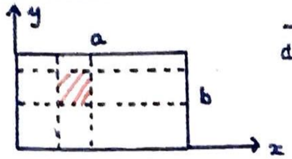
Surface et Volume :

• Rectangle :



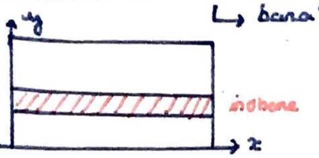
$\vec{dl}\begin{vmatrix} dx \\ dy \\ \cancel{dz} \end{vmatrix}\Big\} d^2S = dx\,dy$

$$S_R = \iint_R d^2S = \int_0^a\int_0^b dx\,dy = \int_0^a dx \int_0^b dy = a\times b$$

$\underline{S_R = a\times b}$

exemple de découpage

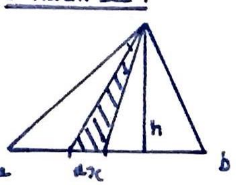
↳ bandelette $P(z)$



$$F = \int_B P(z)\, d^2S \quad \text{avec} \quad d^2z = L\,dz$$

$$\underline{F = \int_0^H P(z)\times L\,dz}$$

• Triangle :



$$d^2S = \frac{dx \cdot h}{2}$$

$$\Rightarrow S = \int_0^H \frac{dx\,h}{2} = \frac{h}{2}\int_0^b dx = \frac{bh}{2}$$

$$\underline{S = \frac{\text{base} \times \text{hauteur}}{2}}$$

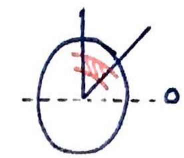
• Disque :

*** cercle**

$\vec{dl} = \begin{vmatrix} \cancel{dr} \\ r\,d\theta \\ \cancel{dz} \end{vmatrix}$

$$P = \int_{\text{cercle}} dl = \int_0^{2\pi} R\,d\theta = R\int_0^{2\pi} d\theta = \underline{2\pi R}$$

*** Disque**



$\vec{dl} = \begin{vmatrix} dR \\ r\,d\theta \\ \cancel{dz} \end{vmatrix} \qquad d^2S = dr\cdot r\,d\theta$

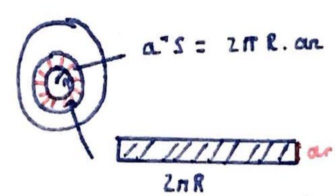
$$S_d = \iint d^2S = \int_0^R\int_0^{2\pi} r\,d\theta\,dr = r\int_0^R dr\int_0^{2\pi} d\theta = 2\pi\left[\frac{r^2}{2}\right]_0^R$$

$\underline{S_d = \pi R^2}$

exemple 1D :

$d^2S = \dfrac{R\times R\,d\theta}{2}$ (triangle)

$$S = \frac{R^2}{2}\int_0^{2\pi} d\theta = \frac{2\pi R^2}{2} = \pi R^2$$



$$S = 2\pi R\,dr = 2\pi\int_0^R r\,dr = 2\pi\left[\frac{R^2}{2}\right]$$

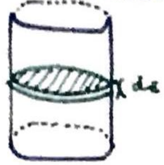
$\underline{= \pi R^2}$

$$D_v = \iint_D v(r)\cdot d^2S = \underline{\int_0^R v(r)\, 2\pi r\,dr}$$

↑ débit volumique

Cylindre :

Surface latérale $d\ell \begin{vmatrix} dr \\ r\,d\theta \\ dz \end{vmatrix}$

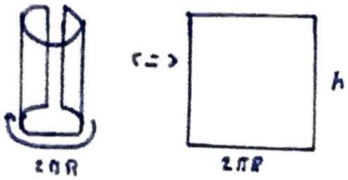


$d^2S = 2\pi R \cdot dz$

$S = \int_0^H d^2S$

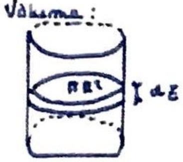
$= 2\pi R \int^H dz$

$\Rightarrow S = 2\pi R H$



soit $S = 2\pi R h$

Volume :



$d^3V = \pi R^2 dz$

$V = \int^H \pi R^2 dz$

$= \pi R^2 \int^H dz$

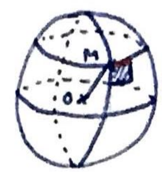
$V = \pi R^2 h$

Application : Soit $\rho(z)$ la masse volumique

$M = \int_0^H dm(z) = \int_0 \rho(z) dV = \int_0^H \rho(z) . S dz$

car $dm(z) = \rho(z) dV$

Sphère :



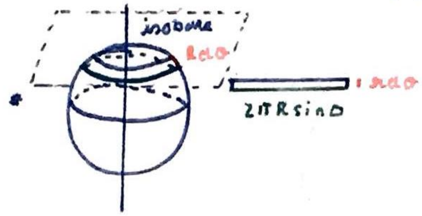
$d\ell : \begin{vmatrix} dr \\ R\,d\theta \\ R\sin\theta\, d\varphi \end{vmatrix}$

$d^2S = R d\theta \times R\sin\theta d\varphi$

$= R^2 \sin\theta\, d\theta\, d\varphi$

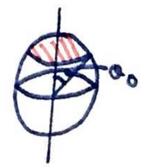
$\theta [0, \pi] \quad \varphi = [0, 2\pi]$

$S = \int^{\pi}\int^{2\pi} R^2 \sin\theta\, d\theta\, d\varphi = R^2 \underbrace{\int^{\pi} \sin\theta\, d\theta}_{[-\cos(\theta)]_0^{\pi}} \int^{2\pi} d\varphi = R^2 \times 2 \times 2\pi$ soit $S = 4\pi R^2$



soit $d^2S : 2\pi R \sin\theta \times r d\theta$

exemple $dF = P(z) \times d^2S \times \cos\theta \Rightarrow F = \int^{\pi} P(z) \times d^2S \times \cos\theta\, d\theta$



$S_c = \int_0^{\theta_0} d^2V = \int_0^{\theta_0} 2\pi R^2 \sin\theta\, d\theta = 2\pi R^2 [-\cos(\theta)]_0^{\theta_0}$

$S = 2\pi R^2 [1 - \cos(\theta_0)]$

$\theta_0 = 0 \quad S = 0 \quad \checkmark$

$\theta_0 = \pi \quad S = 4\pi R^2 \quad \checkmark$

Volume

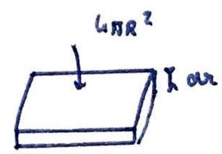
$dV = dr \times r d\theta \times r\sin\theta\, d\varphi \Rightarrow V = \int_0^R\int_0^{\pi}\int_0^{2\pi} r^2 dr \sin\theta\, d\theta\, d\varphi$

$[0, R] [0, \pi] [0, 2\pi]$

$= \underbrace{\int_0^R r^2 dr}_{\frac{R^3}{3}} \underbrace{\int_0^{\pi} \sin\theta\, d\theta}_{2} \underbrace{\int_0^{2\pi} d\varphi}_{2\pi}$

$\Rightarrow V = \frac{4}{3}\pi R^3$

$d^3V \rightarrow$ idée



$d^3V = 4\pi R^2 dr \Rightarrow V = \int^R d^3V = \int^R 4\pi R^2 dr = \frac{4}{3}\pi R^3$