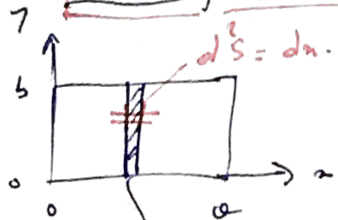
Surfaces et Volumes

Ajouter $d\vec{l} = dr\,\vec{u_r} + r\,d\theta\,\vec{u_\theta} + dz\,\vec{u_z}$

Pour chaque surface : $dS = \int_0^R r\,dr \int_0^{\alpha} d\theta$ etc

Rectangle

$d\vec{l} = \begin{vmatrix} dx \\ dy \\ dz \end{vmatrix}$



$d^2S = dx \cdot dy$

$dS = dx \times b$

Soit $S = \int_0^a dS = \int_0^a b\,dx = b\int_0^a dx = b(a-0)$

$\boxed{S = ab}$

$\hookrightarrow \boxed{F = \int P\,dS \quad \int_0^a P(x)\cdot b\,dx = b\int_0^a P(x)\,dx}$

ex barrage.

Triangle :

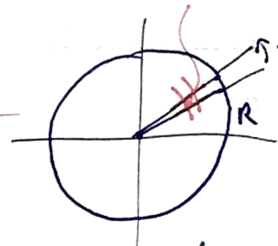
$dS = \frac{h \times dx}{2}$ $\quad S = \frac{h}{2}\int_0^b dx = \frac{bh}{2}$

$\boxed{S = \frac{bh}{2}}$

Disque :

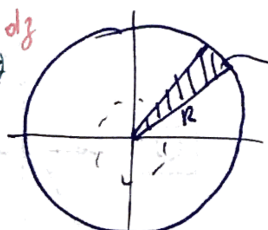
$d\vec{l}\begin{vmatrix} dr \\ r\,d\theta \\ dz \end{vmatrix}$

$d^2S = r\,d\theta \cdot dr$

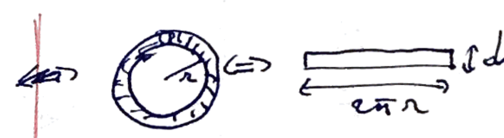


$P = \int_{\text{Cercle}} d^1l$ avec $d^1l = R\,d\theta$ $\quad P = \int_0^{2\pi} R\,d\theta = R\int_0^{2\pi} d\theta$

$P = R \times (2\pi - 0) = 2\pi R$.

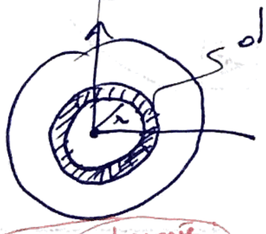


$d^1S = \frac{R\,d\theta \times R}{2}$ $\quad S = \int_0^{2\pi} \frac{R^2\,d\theta}{2} = \frac{R^2}{2}(2\pi - 0) = \pi R^2$



$d^1S \simeq$ périmètre $\times$ épaisseur $= 2\pi r \times dr$

$= 2\pi r\,dr$

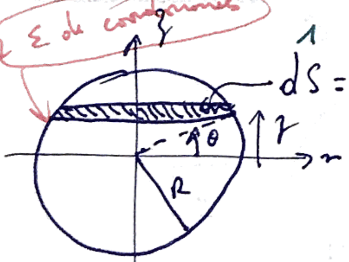


Donc $S = \int_0^R 2\pi r\,dr = 2\pi\int_0^R r\,dr = 2\pi\left(\frac{R^2}{2} - 0\right)$

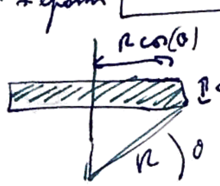
$\boxed{S = \pi R^2}$

Rq : On a pas tjrs le choix de d^1S

(!) 2 Σ de coordonnées



d^1S = longueur $\times$ épaisseur



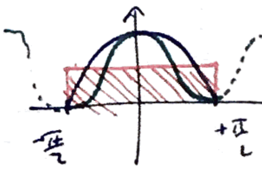
$\boxed{d^1S = 2R\cos\theta \times dz}$

Avec $z = R\sin(\theta)$

$\hookrightarrow dz = R\cos(\theta)\cdot d\theta$.

Soit $d^1S = 2R^2\cos^2(\theta)\,d\theta$.

$S = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} 2R^2\cos^2(\theta)\,d\theta = 2R^2 \underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2(\theta)\,d\theta}_{\pi \times \langle\cos^2(\theta)\rangle = \frac{1}{2}}$



$\Rightarrow S = \frac{\pi}{2} \times 2R^2 = \pi R^2$

$\boxed{S = \pi R^2}$

$\boxed{\text{ex : } F = \int P(z)\,d^1S = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} P(z(\theta)) \cdot 2R\cos(\theta)\cdot dz}$

cf TD

Cylindre :

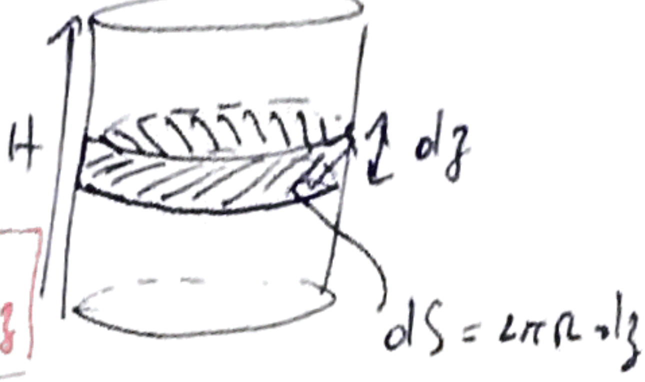
$\circledast$ $S_{sup} = S_{inf} = \pi R^2$ $\quad P = 2\pi R$

$\vec{dl} \begin{vmatrix} dr \\ r\,d\theta \\ dz \end{vmatrix}$

$\rightarrow$ $d^1 S_{lat} = 2\pi R \times dz$

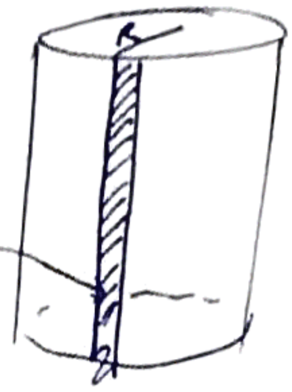
$\hookrightarrow S_{lat} = 2\pi R \int_0^H dz = 2\pi R H$

$\boxed{d^2 S = R\,d\theta . dz}$



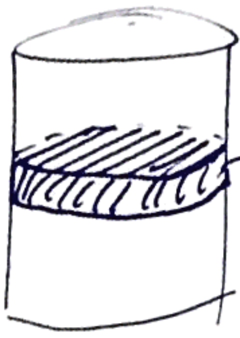
$\rightarrow$ $d^1 S_{lat} = H \times R\,d\theta$

$\hookrightarrow S_{lat} = HR \int_0^{2\pi} d\theta = 2\pi R H$



Volume

$\rightarrow$

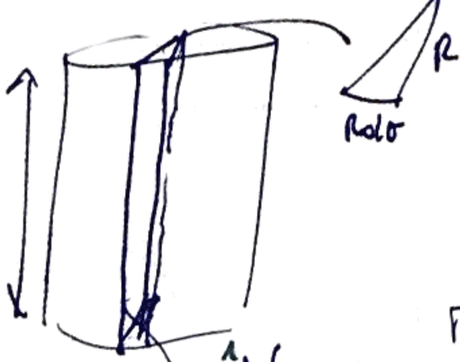


$d^1 V = \overbrace{\pi R^2}^{S} \times dz$

$V = \int_0^H \pi R^2 dz = \pi R^2 H$

ex : $M_{tot} = \int_0^H \pi R^2 \rho(z) . dz$ masse volumique

$\rightarrow$ H



$dS = \frac{R \times R d\theta}{2}$

$d^1 V = H \times \overbrace{\frac{R d\theta \times R}{2}}^{S_{triang}}$

$V = \frac{HR^2}{2} \int_0^{2\pi} d\theta = \frac{2\pi}{2} R^2 H = \pi R^2 H$.

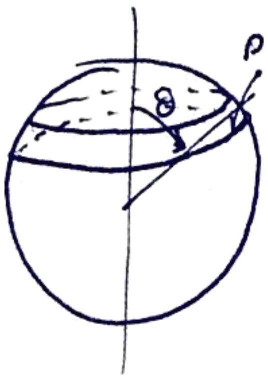
$\vec{dl} \begin{vmatrix} dr \\ r\,d\theta \\ r\sin(\theta)\, d\varphi \end{vmatrix}$

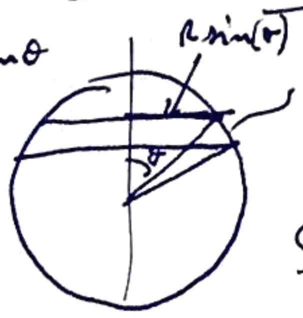
$\boxed{d^2 S = r^2 \sin(\theta) . d\theta . d\varphi}$

⚠ Savoir le retrouver géom

Sphère :

$\circledast$





$\boxed{d^1 S = P \times R\,d\theta = 2\pi R^2 \sin(\theta) d\theta}$

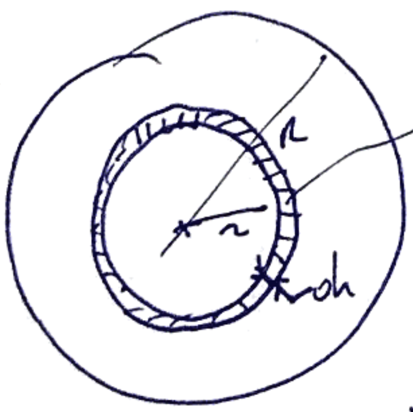
$S = 2\pi R^2 \int_0^{\pi} \sin(\theta) . d\theta$

$S = 2\pi R^2 \left[-\cos(\theta)\right]_0^{\pi} = 2\pi R^2 \left[\underbrace{1 - (-1)}_{2}\right] = 4\pi R^2$

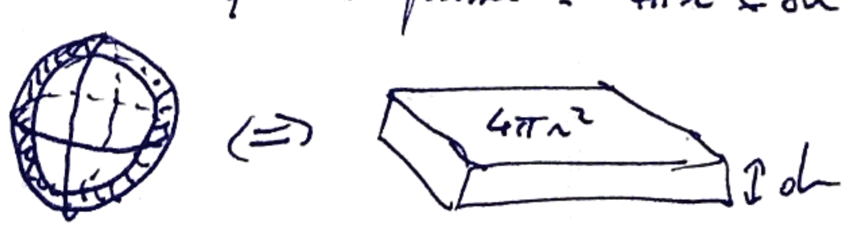
$\boxed{S_{sphère} = 4\pi R^2}$

ex : TD hémisphère de Magdebourg. $F_x = \int \underline{2\pi R^2 \sin(\theta) \times} P_0 \cos(\theta) d\theta$. $\quad P_0 \vec{dS} . \vec{e_x}$

Boule :



coquille : $dV = S_{sphère} \times$ épaisseur $= 4\pi r^2 \times dr$



$V = \int_0^R 4\pi r^2 dr = 4\pi \int_0^R r^2 dr = 4\pi \left(\frac{R^3}{3} - 0\right) = \frac{4\pi}{3} R^3$

Rq : $V(r+dr) - V(r) = \frac{4\pi}{3}\left[(r+dr)^3 - r^3\right] = \frac{4\pi}{3} \frac{dr^3}{dr} . dr = \frac{4\pi}{3} 3 r^2 dr = 4\pi r^2 dr$.

Disque : $\pi R^2 \xrightarrow{\text{dérivée}} 2\pi R$

boule : $\frac{4\pi R^3}{3} \rightarrow 4\pi R^2$

$S(\theta) = 2\pi R^2 [1 - \cos(\theta)]$ $\quad$ À faire